Hale School

Mathematics Specialist

Test 1 --- Term 1 2017

Complex Numbers

/ 45

Name: ______________________________

Instructions:

- **CAS calculators are NOT allowed**
- **External notes are not allowed**
- **Duration of test: 45 minutes**
- **Show your working clearly**
- **Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)**
- **This test contributes to 7% of the year (school) mark**

All arguments must be given using principal values.

Question 1 **(4 marks: 1, 1, 2)**

The following diagram shows a complex number z on the complex plane.

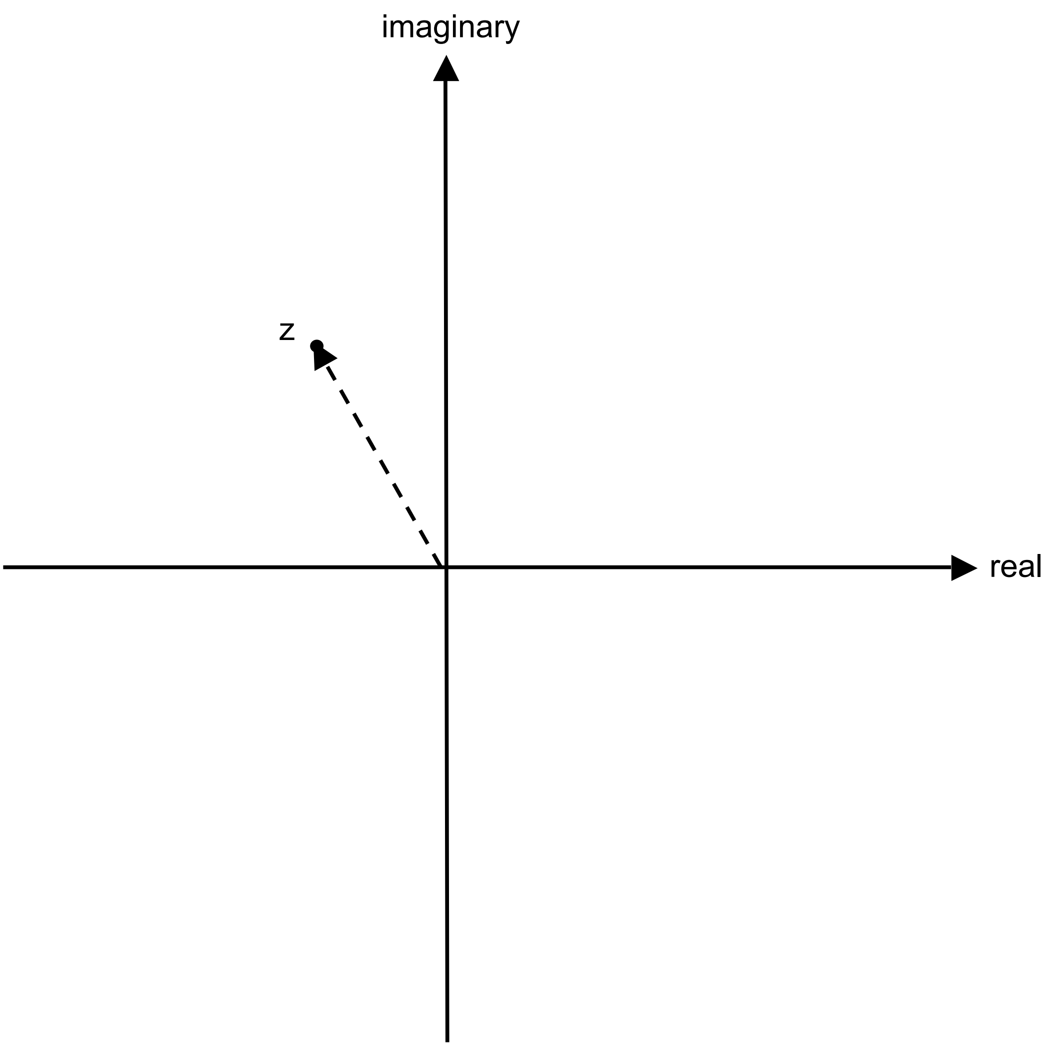
It is known that $|z| = 2$

Locate the following complex numbers. Label your answers clearly.

(a) $z_1 = -2z$ (1 mark)

(b) $z_2 = \bar{z} + 2i$ (1 mark)

(c) $z_3 = \dfrac{\sqrt{2}\, z}{(1+i)}$ (2 marks)



Question 2 (6 marks: 2, 4)

Simplify the following expressions, leaving your answers in rectangular form;

(a) $\dfrac{3+4i}{2-3i}$

(b) $\dfrac{\left(2cis(\pi/8)\right)^4}{\left(\sqrt{2}cis(\pi/4)\right)^5}$

Question 3 **(5 marks)**

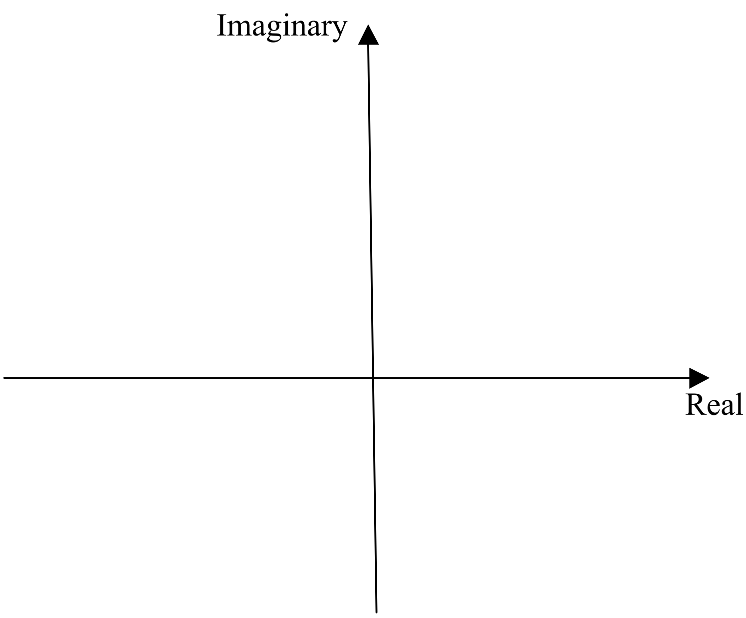
Solve the equation $5 - i = z(3 + 2i) + 3\bar{z}$

Question 4 **(4 marks: 2, 2)**

On the Argand plane below, sketch the locus of points given by

(a) $|z - 2| = |z + 2i|$

(b) $|z - 2| = |z + 2i| - 2\sqrt{2}$



Question 5 (7 marks: 1, 1, 2 and 3)

Consider the polynomial $f(z) = z^5 + z^3 - 8iz^2 - 8i$.

(a) Show that $(z + i)$ is factor of $f(z)$ (1 mark)

(b) Find another factor of $f(z)$ (1 marks)

(c) Factorise $f(z)$ (2 marks)

(d) Solve the equation $f(z) = 0$, giving answers in polar form. (3 marks)

Question 6 (7 marks)

Use De Moivre's Theorem $\left(\cos\theta + i\sin\theta\right)^n = \cos(n\theta) + i\sin(n\theta)$

To prove the trigonometric identity $\sin(3\theta)\cos(2\theta) = 8\sin^5\theta - 10\sin^3\theta + 3\sin\theta$

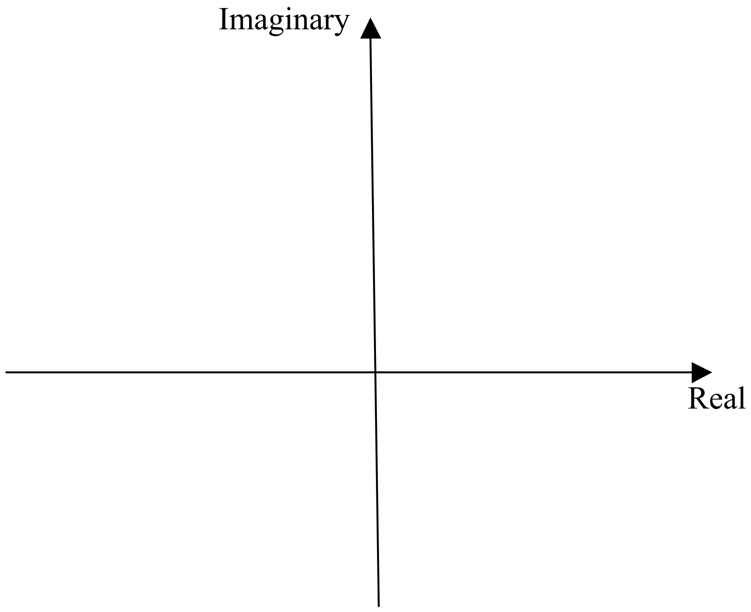
Question 7 (5 marks)

It is known that $(z-2+3i)$ is a factor of $f(z)=z^4-4z^3+9z^2+16z-52$.
Use this information to find all the roots of the equation $f(z)=0$

Question 8 (7 marks: 4, 3)

(a) On the axes below sketch the region of the Argand diagram for which (4 marks)

$-\frac{\pi}{2} \leq \arg(z-1-i) \leq 0 \quad and \quad |z-1-i| \leq \sqrt{2}$



(b) For the region defined in part (a) above, find the minimum and maximum values for

$\tan\theta \quad where\ \theta = \arg(z)$